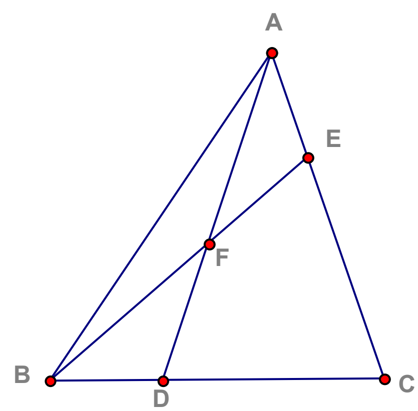
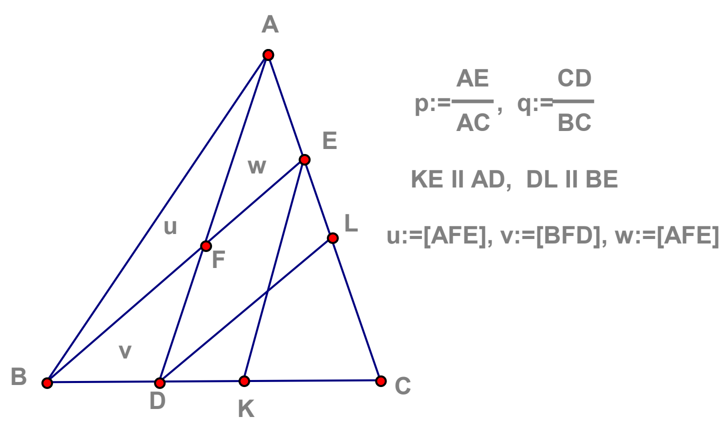
Area via given partial areas.
https://www.linkedin.com/feed/update/urn:li:activity:6714559416168935424
In the figure shown, if the area of triangle ABF is 3 sq. unites, that of triangle AFE is 4 sq. units and that of triangle BFD is 2 sq. units, find the area of quadrilateral $FECD$.



(1) 8 (2) 24 (3) 72 (4) 96.

Solution by Arkady Alt, San Jose, California, USA.



Noting that $\dfrac{BF}{FE}=\dfrac{u}{w}$ and $\dfrac{AF}{FD}=\dfrac{u}{v}$ we obtain $\dfrac{BD}{DK}=\dfrac{BF}{FE}=\dfrac{u}{w}$ and $\dfrac{AE}{EL}=\dfrac{AF}{FD}=\dfrac{u}{v}$.
From the other hand since $\dfrac{BD}{BC}=1-q$ and $\dfrac{DK}{DC}=\dfrac{AE}{AC}=p$ then $\dfrac{DK}{BD}=\dfrac{p\cdot CD}{BD}=\dfrac{p\cdot q\cdot BC}{(1-q)BC}=\dfrac{p\cdot q}{1-q}$. Also, since $\dfrac{AE}{AC}=p, \dfrac{CE}{AC}=1-p \Leftrightarrow CE=(1-p)AC$ and $\dfrac{EL}{CE}=\dfrac{BD}{BC}=1-q \Leftrightarrow EL=(1-q)CE=(1-q)(1-p)AC$ we obtain $\dfrac{EL}{AE}=\dfrac{(1-q)(1-p)AC}{pAC}=\dfrac{(1-q)(1-p)}{p}$.
From the other hand we have $\dfrac{DK}{BD}=\dfrac{FE}{FB}=\dfrac{w}{u}, \dfrac{EL}{AE}=\dfrac{v}{u}$. Hence,
$\dfrac{p\cdot q}{1-q}=\dfrac{w}{u}$ and $\dfrac{(1-q)(1-p)}{p}=\dfrac{v}{u}$.

Since $\frac{p \cdot q}{1-q} \cdot \frac{(1-q)(1-p)}{p} = \frac{vw}{u^2} \Leftrightarrow q(1-p) = \frac{vw}{u^2}$ and $\frac{p \cdot q}{1-q} = \frac{w}{u} \Leftrightarrow p = \frac{w(1-q)}{qu}$

we obtain $q\left(1 - \frac{w(1-q)}{qu}\right) = \frac{vw}{u^2} \Leftrightarrow q - \frac{w(1-q)}{u} = \frac{vw}{u^2} \Leftrightarrow q(u+w) = \frac{vw}{u} + w \Leftrightarrow$

$q = \frac{w(u+v)}{u(u+w)} \Rightarrow p = \frac{w\left(1/\left(\frac{w(u+v)}{u(u+w)}\right) - 1\right)}{u} = \frac{u^2 - vw}{u(u+v)}.$

Since $\frac{[BFC]}{[AFB]} = \frac{1-p}{p} \Leftrightarrow [BFC] = u \cdot \frac{1-p}{p} = u \cdot \frac{1 - \frac{u^2 - vw}{u(u+v)}}{\frac{u^2 - vw}{u(u+v)}} = \frac{uv(u+w)}{u^2 - vw}$

and $\frac{[AFC]}{[AFB]} = \frac{q}{1-q} \Leftrightarrow [AFC] = u \cdot \frac{q}{1-q} = u \cdot \frac{\frac{w(u+v)}{u(u+w)}}{1 - \frac{w(u+v)}{u(u+w)}} = \frac{uw(u+v)}{u^2 - vw}.$

Hence $[ABC] = [AFC] + [BFC] + [AFB] = u + \frac{uw(u+v)}{u^2 - vw} + \frac{uv(u+w)}{u^2 - vw} = \frac{u(u+w)(u+v)}{u^2 - vw}$

and, therefore, $[DFEC] = \frac{u(u+w)(u+v)}{u^2 - vw} - u - v - w = \frac{vw(2u+v+w)}{u^2 - vw}.$

In particular for $u = 3, w = 4, v = 2$ we obtain $[DFEC] = \frac{2 \cdot 4(2 \cdot 3 + 2 + 4)}{3^2 - 2 \cdot 4} = 96.$